An Overview of Weak Gravitational Lensing

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Abstract

Weak gravitational lensing, or the subtle distortion of light by massive structures, is an important tool to studying to modern astrophysics. It enables mapping of dark matter, the study of galaxy clusters, and the refinement of cosmological models. This paper explores the history, current advancements, and open questions in the field, with insights from Dr. Ian Dell'Antonio's research on spectrotomography and the LoVoCCS survey. There is potential in weak lensing to transform our understanding of the universe's structure and evolution.

1 Introduction

Weak gravitational lensing refers to small distortions in the shapes of background galaxies, caused by the gravitational potential of intervening mass distributions. Unlike strong lensing, which produces big dramatic arcs and multiple images, weak lensing (WL) requires statistical analysis of large galaxy samples to find patterns of mass distribution.

This has become significant in modern astrophysics, offering insight into the invisible dark matter that makes up the majority of the universe's mass, enabling detailed mapping of the cosmic web.

Recent advancements in observational techniques, including spectroscopic redshifts and innovative surveys like the Local Volume Complete Cluster Survey (LoVoCCS), have improved the precision of weak lensing, which can help us understand unsolved questions about the evolution of the universe.

This paper explores the scientific significance of weak gravitational lensing, examines recent advancements in the field, and highlights current challenges and questions through the lens of expert insights from Dr. Ian Dell'Antonio of Brown University.

2 Background

2.1 History of Weak Lensing

In this section I will be paraphrasing section 1 of Bartelmann and Maturi [2016], where Bartelmann and Maturi gave a great overview of the history of Weak Lensing (WL).

The concept of gravity affecting light dates back to the early foundations of physics. In 1704, Sir Isaac Newton speculated that light could be deflected by massive objects, suggesting an effect on light by gravity in his Opticks, Newton et al. [1721]. However, Newtonian physics could not fully address this phenomenon, as light was considered massless, raising questions about how gravity could act on it.

The first quantitative treatment of light deflection was carried out in an article in 1801 by Johann Georg von Soldner, von Soldner [1801]. Soldner applied Newtonian principles to calculate the deflection angle for a light ray grazing the Sun. While his calculations were insightful, they underestimated the role of spacetime curvature, especially since Newtonian physics lacked the framework to account for it.

Einstein's theory of general relativity was a significant leap in the idea of gravity affecting light. In 1911, Einstein predicted a deflection angle similar to Soldner's calculations. However, by 1916, he revised his prediction, recognizing the effects of both spatial and temporal curvature of spacetime. This corrected prediction, 1.7 arcseconds for light passing near the Sun, was experimentally confirmed during the 1919 solar eclipse by Arthur Eddington and colleagues, cementing gravitational lensing as a profound confirmation of general relativity.

In 1936, requested by Rudi Mandl, Einstein published on the lensing of a distant star by another star. Around the same time, Fritz Zwicky proposed that galaxy clusters could act as gravitational lenses, laying the groundwork for applying lensing in astrophysical contexts.

The first observational confirmation of lensing by a galaxy was achieved in 1979 (figure 1), marking the dawn of gravitational lensing as an observational tool. Over the next decades, weak lensing became more recognized as a technique for studying the distribution of dark matter and large-scale structure in the universe.

2.2 The Physics of Weak Lensing

2.2.1 Mathematical Framework

Weak gravitational lensing finds its foundation in general relativity, where spacetime curvature governs light propagation. We begin with Einstein's field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

For weak gravitational fields, we consider perturbations about the Minkowski metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ represents metric perturbations. For a static Newtonian potential Φ, the perturbed metric becomes:

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 + \left(1 - \frac{2\Phi}{c^2}\right) \delta_{ij} dx^i dx^j$$

Light follows null geodesics determined by:

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

where $\Gamma^\mu_{\alpha\beta}$ are Christoffel symbols. For weak fields, the transverse deflection angle $\hat{\alpha}$ integrates the gravitational potential gradient:

$$\hat{\alpha} = \frac{2}{c^2}\int \nabla_\perp \Phi \, dl$$

The relationship between true source position β and observed position θ is given by:

$$\beta = \theta - \frac{D_{ds}}{D_s}\hat{\alpha}(D_d\theta)$$

where D_d, D_s, and D_{ds} are angular diameter distances to the lens, source, and between them respectively.

The Jacobi matrix A_{ij} describes image deformation:

$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j}$$

Decomposing into convergence κ and shear $\gamma = \gamma_1 + i\gamma_2$:

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

The convergence relates to surface mass density Σ through:

$$\kappa(\theta) = \frac{\Sigma(D_d\theta)}{\Sigma_{\text{crit}}}, \quad \Sigma_{\text{crit}} = \frac{c^2}{4\pi G}\frac{D_s}{D_d D_{ds}}$$

Shear components derive from the lensing potential ψ:

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}), \quad \gamma_2 = \psi_{,12}$$

where ψ satisfies the 2D Poisson equation:

$$\nabla^2\psi = 2\kappa$$

The complete description involves the optical tidal matrix:

$$\mathcal{T}_{ij} = -\frac{1}{2}\frac{\partial^2\Phi}{\partial x^i \partial x^j}$$

which governs light bundle deformation through the geodesic deviation equation. In cosmology, this incorporates both matter perturbations and background expansion through the Weyl potential.

This framework reveals weak lensing as a direct probe of spacetime geometry shaped by all mass-energy components, including dark matter. For further mathematical details, see Bartelmann and Maturi [2016].

2.2.2 Techniques and Applications

Since weak lensing integrates the effects of all intervening matter in the foreground of the source it can be used to make a cumulative measure of the mass distribution of the universe. This phenomena, called line-of-sight effects, is particularly sensitive to dark matter. This is because dark matter makes up the majority of the universe's mass. This means WL allows researchers to map the dark matter distribution in great detail.

Spectrotomography, a recent advancement in WL research, leverages precise spectroscopic redshift measurements to overcome limitations of traditional photometric redshift. This technique allows for higher resolution of the three-dimensional distribution of matter. See the interview section with Professor Dell'Antonio, where he discusses his research, specifically Dell'Antonio et al. [2020] and Fu et al. [2024], for a more detailed account of this technique and it's difference to photometric redshift

Weak Lensing combined with other observational techniques allow for us to test cosmological models and refine predictions about the fate of the universe. See Fu et al. [2024] for more information on cosmological implications.

The techniques developed for weak lensing extend beyond traditional astrophysics. Spectrotomographic methods, as demonstrated by Dell'Antonio's team, bridge observational and computational astrophysics by integrating large-scale simulations and advanced redshift measurements. Weak lensing studies of dark matter distribution have implications for particle physics, as they offer indirect constraints on dark matter particle properties and interactions

3 Current Research

3.1 LoVoCCS and Spectrotomography

The Local Volume Complete Cluster Survey (LoVoCCS) and spectrotomography stands out as transformative approaches. These methods have significantly improved the precision of weak lensing measurements and opened new avenues for probing the dark matter distribution and galaxy cluster dynamics.

3.1.1 Local Volume Complete Cluster Survey

LoVoCCS is a deep imaging survey designed to map the weak lensing signals of galaxy clusters within a local volume. A key focus of this survey is reconstructing the mass distributions of nearby clusters, such as Abell 2029, which has been extensively studied by Dr. Ian Dell'Antonio and his collaborators. By leveraging advanced weak lensing techniques, LoVoCCS provides high-resolution maps of the dark matter profiles in these clusters, offering insights into their substructures and total mass. These observations are critical for comparing the distribution of dark matter with the baryonic matter traced by X-ray gas and galaxies, helping to test and refine theoretical models of cluster formation and evolution.

One of the survey's key achievements has been its ability to confirm the spatial alignment between the cluster's dark matter halo and its brightest central galaxy. Such studies illuminate the physical processes governing galaxy-cluster interactions and contribute to our understanding of the co-evolution of galaxies and their dark matter environments.

3.1.2 Spectrotomography

Spectrotomography is a revolutionary technique traditional weak lensing studies relying on photometric redshifts, which are estimated from galaxy colors and are prone to errors. Spectrotomography, however, uses spectroscopic redshifts derived from precise spectral line measurements to improve depth and accuracy in three-dimensional mass reconstructions. By providing higher resolution in redshift space, spectrotomography enables more detailed separation of lensing contributions from different structures along the line of sight. This results in a more refined understanding of the distribution and properties of dark matter.

Dr. Dell'Antonio's work in applying spectrotomography to weak lensing, particularly in LoVoCCS, demonstrates its advantages over traditional methods. For example, the weak lensing mass reconstruction of Abell 2029 achieved with spectrotomography reveals unprecedented details of its mass distribution, offering new insights into the cluster's dynamics and its surrounding cosmic web. Moreover, spectrotomography facilitates the identification of secondary structures, such as infalling groups or nearby subclusters, which are otherwise difficult to detect with photometric techniques.

3.1.3 My Research

My work with Professor Källan Berglund at WPI, in collaboration with Northeastern University, focuses on advancing weak lensing techniques through the analysis of SuperBIT telescope data. Professor Jacqueline McCleary's team has developed a gravitational lensing pipeline that processes raw observational data into mass density estimates, and I contribute by testing this pipeline on real datasets. This collaboration has provided me with hands-on experience in weak lensing analysis, from data reduction to mass reconstruction.

A key component of my work involves developing visualization tools for astronomical data formats. I created a repository to process and display FITS (Flexible Image Transport System) files, the standard format for telescope images. These files contain multidimensional arrays encoding observational data alongside critical metadata like telescope pointing coordinates and instrument parameters. Figure 1 shows our coadded image of galaxy cluster PLCKG287d0p32d9, created by combining thirty 20-second SuperBIT exposures through coaddition – a process that enhances resolution by stacking multiple observations.

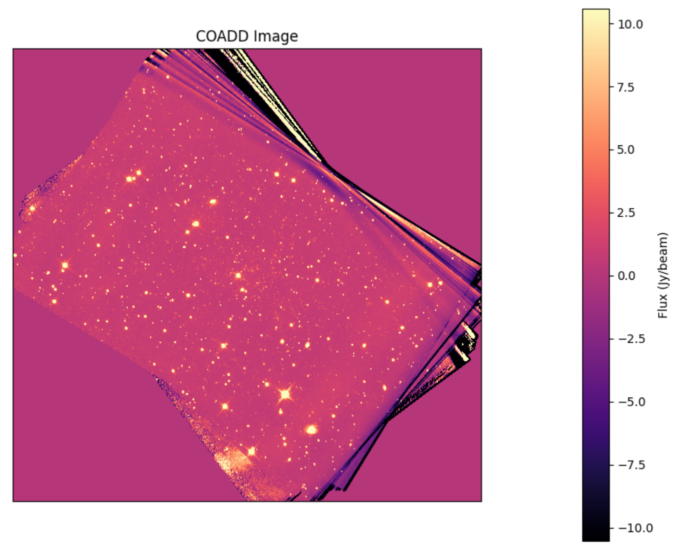

Figure 1. Coadded optical image of PLCKG287d0p32d9 from SuperBIT observations.

Applying the SuperBIT lensing pipeline to this data produces the mass distribution map shown in Figure 2. This "E-mode" map, following the formalism in McCleary et al. [2023], represents the detectable lensing signal component, while the accompanying signal-to-noise ratio (SNR) maps quantify confidence in these mass estimates across the field.

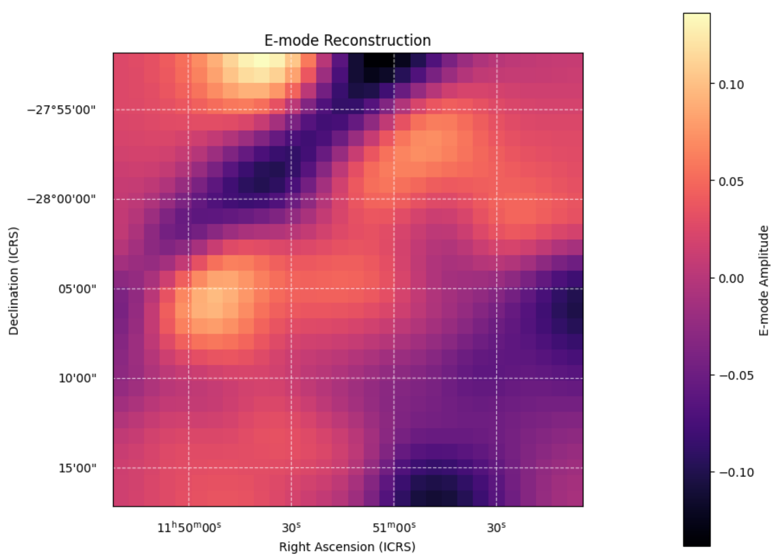

Figure 2. E-mode mass reconstruction of PLCKG287d0p32d9 with SuperBIT data.

My future work aims to enhance these visualizations by incorporating Right Ascension and Declination coordinates from the FITS headers, currently limited by coordinate system parsing challenges. Additionally, I plan to overlay the mass map with the coadded optical image to investigate correlations between matter distributions and baryonic matter visible in the cluster's galaxies and intracluster light, directly giving information on where the dark matter is located. The group looks to write a proposal for Hubble observation time in order to get even better images to process.

3.2 Challenges and Advances

Weak lensing signals are subtle, often requiring the analysis of small distortions in galaxy shapes. Accurately disentangling these signals from noise, intrinsic galaxy alignments, and observational systematics is a major challenge. Errors in galaxy shape measurements or biases in photometric redshift estimates can significantly affect the interpretation of weak lensing data. For example, photometric redshift uncertainties introduce scatter that reduces the precision of three-dimensional mass reconstructions.

Weak lensing integrates mass along the entire line of sight, making it difficult to isolate the contributions of individual structures. This blending of signals complicates the interpretation of weak lensing maps and can mask substructures within galaxy clusters.

Modern weak lensing surveys, such as those conducted by the Vera C. Rubin Observatory and the Euclid mission, generate vast amounts of data. Analyzing these datasets requires advanced computational methods to handle both the data volume and the complexity of cosmological simulations.

Weak lensing analyses rely on assumptions about the underlying cosmological model, such as the ΛCDM framework. Deviations from this model, or uncertainties in key parameters like the dark energy equation of state, can introduce biases in the interpretation of weak lensing results.

One of the most significant advancements in weak lensing research is the adoption of spectrotomography. By leveraging precise spectroscopic redshifts, this technique minimizes uncertainties associated with photometric methods, enabling more accurate three-dimensional mass reconstructions. As demonstrated by Dell'Antonio and collaborators in the LoVoCCS survey, spectrotomography has revealed detailed substructures within galaxy clusters like Abell 2029, offering insights into their internal dynamics and evolution.

Advanced algorithms now allow for more precise measurements of galaxy shapes, correcting for systematics like point-spread function (PSF) distortions. Methods such as forward modeling and machine learning-based approaches have significantly enhanced the robustness of shape measurements, a cornerstone of weak lensing analysis.

The advent of next-generation telescopes and surveys has dramatically increased the scope of weak lensing studies. Instruments like the Vera C. Rubin Observatory and ESA's Euclid telescope offer deeper, wider, and higher-resolution imaging, enabling the detection of weak lensing signals across larger cosmic volumes.

Integrating weak lensing observations with advanced cosmological simulations has improved our understanding of the relationship between dark matter and galaxies. These combined approaches allow researchers to test theoretical predictions against real-world data, refining models of structure formation and galaxy evolution.

4 Interview with Dr. Ian Dell'Antonio

I first want to thank Dr. Ian Dell'Antonio for the interview. Professor Dell'Antonio provided valuable insights into the field of weak gravitational lensing during our interview, highlighting both the fundamental principles and the challenges of this methodology.

4.1 The Basics of Weak Lensing

Dr. Dell'Antonio explained that weak gravitational lensing leverages the distortion of background galaxies caused by the mass of foreground objects. This technique measures the distribution and amount of mass in the universe by statistically analyzing the orientation of numerous galaxies. While the true shape of any individual galaxy is unknown, the average orientation of a large sample provides a null measurement. Gravitational lensing introduces a "bullseye" pattern of alignment in the background galaxies' shapes, linearly proportional to the intervening mass. Unlike strong lensing, weak lensing requires statistical analysis of many objects, as individual distortions are too subtle to reconstruct directly.

4.2 Challenges in Measurement

Dr. Dell'Antonio discussed two significant sources of uncertainty, shape noise and photometric redshift errors. Shape noise arises from the intrinsic random orientation of galaxies, which complicates measurements. This uncertainty can be mitigated by increasing the sample size of galaxies. Photometric redshift errors stem from estimating distances using galaxy colors rather than precise spectroscopic measurements. Calibration against known spectroscopic redshifts helps refine these estimates, but the error margins remain larger than those in spectroscopic methods.

4.3 LoVoCCS and Spectrotomography

Dr. Dell'Antonio highlighted the Local Volume Complete Cluster Survey (LoVoCCS), which incorporates spectrotomography to improve weak lensing analysis. Spectrotomography utilizes precise spectroscopic redshifts to calculate the distortion's dependence on the distances of background galaxies. This method allows researchers to observe the predicted relationship between distortion strength and distance, a critical test for models of the universe's expansion. However, the limited number of spectroscopic measurements presents a significant challenge. Despite this, the survey provides valuable insights and demonstrates the potential of combining large datasets with spectroscopic precision.

4.4 Future Directions in Weak Lensing

The field's primary goal, Dr. Dell'Antonio noted, is to measure more objects with greater accuracy. Initiatives like the Rubin Observatory, Euclid satellite, and Roman Space Telescope aim to achieve this by conducting large-scale surveys. Additionally, efforts are underway to refine shape measurements and mitigate distortions caused by atmospheric and instrumental effects. Space-based observations, which avoid atmospheric interference, are a promising avenue for sharper images and better measurements. Long-term, weak lensing serves as a tool for mapping dark matter and understanding cosmic structure growth, offering insights into dark energy and the universe's expansion.

5 Conclusion

Weak gravitational lensing is a powerful tool for uncovering the universe's structure, offering insights into dark matter and cosmic evolution. By analyzing subtle distortions in galaxy shapes, researchers construct mass maps and explore the dynamics of dark energy. Despite challenges like shape noise and photometric redshift errors, advancements such as spectro-tomography and next-generation surveys promise to refine measurements and expand our understanding. As highlighted by Dr. Ian Dell'Antonio, the integration of innovative techniques and expanded data will drive transformative discoveries, solidifying weak lensing's role in modern astrophysics.

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In this paper, ChatGPT was utilized for formatting in LaTeX, implementing BibTeX, and generating summaries of the cited texts. AI was not used to generate any of the writing.

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